

Solution Steps

Maple 2021 includes numerous new algorithms for showing step-by-step solutions for a variety of problems in mathematics.

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Long Division

The [LongDivision](#) command gives a visual solution to an arithmetic or polynomial long division problem, showing all of the intermediate steps.

> *with(Student-Basics)* :

> *LongDivision*($48x^4 + 284x^3 + 620x^2 + 593x + 210, 2x + 3$)

$$\begin{array}{r} 24x^3 + 106x^2 + 151x + 70 \\ 2x + 3 \overline{) 48x^4 + 284x^3 + 620x^2 + 593x + 210} \\ \underline{48x^4 + 72x^3} \\ 212x^3 + 620x^2 \\ \underline{212x^3 + 318x^2} \\ 302x^2 + 593x \\ \underline{302x^2 + 453x} \\ 140x + 210 \\ \underline{140x + 210} \\ 0 \end{array}$$

(1.1)

> `LongDivision(1001, 30, 'decimaldigits'=4);`

$$\begin{array}{r} \underline{33.3666} \\ 30 \) \ 1001.0000 \\ \underline{90} \\ 101 \\ \underline{90} \\ 110 \\ \underline{90} \\ 200 \\ \underline{180} \\ 200 \\ \underline{180} \\ 200 \\ \underline{180} \\ 20 \end{array}$$

(1.2)

Factoring

The [FactorSteps](#) command shows the steps in factoring a polynomial.

> `with(Student-Basics) :`

> `FactorSteps($x^3 + 6x^2 + 12x + 8$)`

$$x^3 + 6x^2 + 12x + 8$$

▫ 1. Trial Evaluations

- Rewrite in standard form

$$x^3 + 6x^2 + 12x + 8$$

- The factors of the constant coefficient 8 are:

$$C = \{1, 2, 4, 8\}$$

- Trial evaluations of x in $\pm C$ find $x = -2$ satisfies the equation, so $x + 2$ is a factor

$$(x^3 + 6x^2 + 12x + 8) \Big|_{x=-2} = 0$$

- Divide by $x + 2$

$$\begin{array}{r}
 x^2 + 4x + 4 \\
 x + 2 \overline{) x^3 + 6x^2 + 12x + 8} \\
 \underline{x^3 + 2x^2} \\
 4x^2 + 12x \\
 \underline{4x^2 + 8x} \\
 4x + 8 \\
 \underline{4x + 8} \\
 0
 \end{array}$$

- Quotient times divisor from long division

$$(x^2 + 4x + 4) \cdot (x + 2)$$

• 2. Examine term:

$$x^2 + 4x + 4$$

▫ 3. Apply the AC Method

- Examine quadratic

$$(x^2 + 4x + 4)$$

- Look at the coefficients, $Ax^2 + Bx + C$

$$["A" = 1, "B" = 4, "C" = 4]$$

- Find factors of $|AC| = |1 \cdot 4| = 4$

$$\{1, 2, 4\}$$

- Find pairs of the above factors, which, when multiplied equal 4

$$\{1 \cdot 4, 2 \cdot 2\}$$

- Which pairs of these factors have a sum of $B = 4$? Found:

(2.1)

Solve

The [SolveSteps](#) command shows the steps in solving an equation or system of equations

> *with(Student:-Basics)* :

> *SolveSteps*($5 e^{4x} = 16$)

Let's solve

$$5 \cdot e^{4x} = 16$$

▫ Convert from exponential equation

◦ Divide both sides by 5

$$\frac{5 \cdot e^{4x}}{5} = \frac{16}{5}$$

◦ Simplify

$$e^{4x} = \frac{16}{5}$$

◦ Apply ln to each side

$$\ln(e^{4x}) = \ln\left(\frac{16}{5}\right)$$

◦ Apply ln rule: $\ln(e^b) = b$

$$4x = \ln\left(\frac{16}{5}\right)$$

• Divide both sides by 4

$$\frac{4x}{4} = \frac{\ln\left(\frac{16}{5}\right)}{4}$$

• Exact solution

$$x = \frac{\ln\left(\frac{16}{5}\right)}{4}$$

• Approximate solution

$$x = 0.2907877025$$

(3.1)

> *SolveSteps*($[12x + y = 18, 7x - 8y = 32]$)

Let's solve

$$[12 \cdot x + y = 18, 7 \cdot x - 8 \cdot y = 32]$$

- Pick the 2nd equation to solve for y

$$7 \cdot x - 8 \cdot y = 32$$

- A: isolate for y

- Subtract $7 \cdot x$ from both sides

$$7 \cdot x - 8 \cdot y - 7 \cdot x = 32 - 7 \cdot x$$

- Simplify

$$(-8) \cdot y = 32 - 7 \cdot x$$

- Divide both sides by -8

$$\frac{(-8) \cdot y}{-8} = \frac{32 - 7 \cdot x}{-8}$$

- Simplify

$$y = -4 + \frac{7x}{8}$$

- Solution

$$y = -4 + \frac{7x}{8}$$

- Substitute the value of $y = -4 + \frac{7x}{8}$ into the 1st equation of the system

$$12 \cdot x + \left(-4 + \frac{7x}{8}\right) = 18$$

- Solve for x

- Evaluate subtraction and addition

$$\frac{103x}{8} - 4 = 18$$

- Add 4 to both sides

$$\frac{103}{8} \cdot x - 4 + 4 = 18 + 4$$

- Simplify

$$\frac{103}{8} \cdot x = 22$$

(3.2)

- Divide both sides by $\frac{103}{8}$

$$\frac{x \cdot \left(\frac{103}{8}\right)}{\frac{103}{8}} = \frac{22}{\frac{103}{8}}$$

Calculus: Integration, Differentiation, and Limits

The [ShowSolution](#) command has been improved to show more detailed steps when solving integration, differentiation, and limit problems.

> *with(Student-Calculus1) :*

> *ShowSolution* $\left(\int \sin(x)^2 dx \right)$

Integration Steps

$$\int \sin(x)^2 dx$$

- 1. Rewrite
 - Equivalent expression

$$\sin(x)^2 = \frac{1}{2} - \frac{\cos(2x)}{2}$$

This gives:

$$\int \left(\frac{1}{2} - \frac{\cos(2x)}{2} \right) dx$$

- 2. Apply the **sum** rule
 - Recall the definition of the **sum** rule

$$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

$$f(x) = \frac{1}{2}$$

$$g(x) = -\frac{\cos(2x)}{2}$$

This gives:

$$\int \frac{1}{2} dx + \int -\frac{\cos(2x)}{2} dx$$

- 3. Apply the **constant** rule to the term $\int \frac{1}{2} dx$
 - Recall the definition of the **constant** rule

$$\int C dx = Cx$$

- This means

$$\int \frac{1}{2} dx = \frac{x}{2}$$

We can now rewrite the integral as:

$$\frac{x}{2} + \int -\frac{\cos(2x)}{2} dx$$

- 4. Apply the **constant multiple** rule to the term $\int -\frac{\cos(2x)}{2} dx$

- Recall the definition of the **constant multiple** rule

$$\int C f(x) dx = C \left(\int f(x) dx \right)$$

- This means:

> ShowSolution(Limit($\frac{\sin(x)}{x}$, $x=0$))

Limit Steps

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$$

1. Apply **L'Hôpital's Rule** rule

$$\lim_{x \rightarrow 0} \cos(x)$$

1. Apply **L'Hôpital's Rule** rule

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{\frac{d}{dx} f(x)}{\frac{d}{dx} g(x)}$$

Recall the definition of the **L'Hôpital's Rule** rule

1 2. Evaluate the limit of $\cos(x)$

$$\frac{\sin(x)}{x} = \cos(x) \quad \text{Rule applied}$$

2. Evaluate the limit of $\cos(x)$

$$\lim_{x \rightarrow a} \cos(f(x)) = \cos(\lim_{x \rightarrow a} f(x))$$

Recall the definition of the **cos** rule

> Diff($x^2 \cdot \sin(x)$, x)

$$\frac{d}{dx} (x^2 \sin(x)) \tag{4.2}$$

> ShowSolution((4.2))

Differentiation Steps

$$\frac{d}{dx} (x^2 \sin(x))$$

$$\frac{d}{dx} (x^2) \sin(x) + x^2 \frac{d}{dx} \sin(x)$$

1. Apply **product** rule

$$2x \sin(x) + x^2 \frac{d}{dx} \sin(x)$$

2. Apply **power** rule

$$2x \sin(x) + x^2 \cos(x)$$

3. Evaluate the derivative of $\sin(x)$

1. Apply **product** rule

$$\frac{d}{dx} (f(x) g(x)) = \frac{d}{dx} f(x) g(x) + f(x) \frac{d}{dx} g(x)$$

Recall the definition of the **product** rule

$$f(x) = x^2$$

$$g(x) = \sin(x)$$

2. Apply **power** rule

$$\frac{d}{dx} (x^n) = n x^{n-1}$$

Recall the definition of the **power** rule

3. Evaluate the derivative of $\sin(x)$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

Recall the definition of the **sin** rule

Differential Equations

The [ODESteps](#) command provides detailed steps when solving ordinary differential

equations and systems of ODEs.

> with(Student.-ODEs) :

$$\text{> } ode1 := t^2 (z(t) + 1) + z(t)^2 (t - 1) \left(\frac{d}{dt} z(t) \right) = 0$$

$$ode1 := t^2 (z(t) + 1) + z(t)^2 (t - 1) \left(\frac{d}{dt} z(t) \right) = 0 \quad (5.1)$$

> ODESteps(ode1)

Let's solve

$$t^2 (z(t) + 1) + z(t)^2 (t - 1) \left(\frac{d}{dt} z(t) \right) = 0$$

- Highest derivative means the order of the ODE is 1

$$\frac{d}{dt} z(t)$$

- Separate variables

$$\frac{\left(\frac{d}{dt} z(t) \right) z(t)^2}{z(t) + 1} = - \frac{t^2}{t - 1} \quad (5.2)$$

- Integrate both sides with respect to t

$$\int \frac{\left(\frac{d}{dt} z(t) \right) z(t)^2}{z(t) + 1} dt = \int - \frac{t^2}{t - 1} dt + _CI$$

- Evaluate integral

$$\frac{z(t)^2}{2} - z(t) + \ln(z(t) + 1) = - \frac{t^2}{2} - t - \ln(t - 1) + _CI$$

$$\text{> } ivp2 := \left\{ \frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) - x e^x = 0, \left(\frac{d}{dx} y(x) \right) \Big|_{x=0} = 0, y(0) = 1 \right\}$$

$$ivp2 := \left\{ \frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) - x e^x = 0, \left(\frac{d}{dx} y(x) \right) \Big|_{\{x=0\}} = 0, y(0) = 1 \right\} \quad (5.3)$$

> ODESteps(ivp2)

Let's solve

$$\left\{ \frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) - x e^x = 0, \left(\frac{d}{dx} y(x) \right) \right\}_{\{x=0\}} = 0, y(0) = 1$$

- Isolate 2nd derivative

$$\frac{d^2}{dx^2} y(x) = \frac{d}{dx} y(x) + x e^x$$

- Group terms with $y(x)$ on the lhs of the ODE and the rest on the rhs of the ODE, ODE is linear

$$\frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) = x e^x$$

- Characteristic polynomial of homogeneous ODE

$$r^2 - r = 0$$

- Factor the characteristic polynomial

$$r(r - 1) = 0$$

- Roots of the characteristic polynomial

$$r = (0, 1)$$

- 1st solution of the homogeneous ODE

$$y_1(x) = 1$$

- 2nd solution of the homogeneous ODE

$$y_2(x) = e^x$$

- General solution of the ODE

$$y(x) = _C1 y_1(x) + _C2 y_2(x) + y_p(x)$$

- Substitute in solutions of the homogeneous ODE

$$y(x) = _C1 + _C2 e^x + y_p(x)$$

- Find a particular solution $y_p(x)$ of the ODE

- Use variation of parameters to find y_p here $f(x)$ is the forcing function

$$\left[y_p(x) = -y_1(x) \left(\int \frac{y_2(x) f(x)}{W(y_1(x), y_2(x))} dx \right) + y_2(x) \left(\int \frac{y_1(x) f(x)}{W(y_1(x), y_2(x))} dx \right), f(x) = x e^x \right] \quad (5.4)$$

- Wronskian of solutions of the homogeneous equation

$$W(y_1(x), y_2(x)) = \begin{bmatrix} 1 & e^x \\ 0 & e^x \end{bmatrix}$$

- Compute Wronskian

Matrix Inverse

The [InverseTutor](#) command now has an option to return detailed steps for finding the matrix inverse.

> *with(Student-LinearAlgebra) :*

> $M := \begin{bmatrix} 3 & -3 & 7 \\ 2 & 4 & 8 \\ -2 & -4 & 6 \end{bmatrix} :$

> *InverseTutor(M, output = steps)*

Compute the inverse of this matrix

$$\begin{bmatrix} 3 & -3 & 7 \\ 2 & 4 & 8 \\ -2 & -4 & 6 \end{bmatrix}$$

- Matrix augmented with identity

$$\begin{bmatrix} 3 & -3 & 7 & 1 & 0 & 0 \\ 2 & 4 & 8 & 0 & 1 & 0 \\ -2 & -4 & 6 & 0 & 0 & 1 \end{bmatrix}$$

- Multiply row 1 by $1/3$

$$\begin{bmatrix} 1 & -1 & \frac{7}{3} & \frac{1}{3} & 0 & 0 \\ 2 & 4 & 8 & 0 & 1 & 0 \\ -2 & -4 & 6 & 0 & 0 & 1 \end{bmatrix}$$

- Add -2 times row 1 to row 2

$$\begin{bmatrix} 1 & -1 & \frac{7}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 6 & \frac{10}{3} & -\frac{2}{3} & 1 & 0 \\ -2 & -4 & 6 & 0 & 0 & 1 \end{bmatrix}$$

- Add 2 times row 1 to row 3

$$\begin{bmatrix} 1 & -1 & \frac{7}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 6 & \frac{10}{3} & -\frac{2}{3} & 1 & 0 \\ 0 & -6 & \frac{32}{3} & \frac{2}{3} & 0 & 1 \end{bmatrix}$$

- Multiply row 2 by $1/6$

$$\begin{bmatrix} 1 & -1 & \frac{7}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & \frac{5}{9} & -\frac{1}{9} & \frac{1}{6} & 0 \\ 0 & -6 & \frac{32}{3} & \frac{2}{3} & 0 & 1 \end{bmatrix}$$

- Add 1 times row 2 to row 1

Eigenvalues

The [EigenvaluesTutor](#) command now has an option to return detailed steps for finding Eigenvalues.

> *with(Student:-LinearAlgebra) :*

$$> M := \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix};$$

> *EigenvaluesTutor(M, output = steps)*

Compute the Eigenvalues

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

- Calculate $A = M - t \cdot \text{Id}$

$$\begin{bmatrix} 1-t & 2 & 0 \\ 2 & 3-t & 2 \\ 0 & 2 & 1-t \end{bmatrix}$$

(7.1)

- Find the determinant; this is also called the characteristic polynomial of M.

$$-t^3 + 5t^2 + t - 5$$

- Solve; the eigenvalues are the roots of the characteristic polynomial.

$$\begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

Eigenvectors

The [EigenvectorsTutor](#) command now has an option to return detailed steps for finding Eigenvectors.

> *with(Student:-LinearAlgebra) :*

> $M := \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix} :$

> *EigenvectorsTutor*(M , *output = steps*)

Compute the eigenvectors

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$

▫ Compute the eigenvalues

◦ Calculate $A=M-t \cdot \text{Id}$

$$\begin{bmatrix} 1-t & 2 & 0 \\ 2 & 3-t & 2 \\ 0 & 2 & 1-t \end{bmatrix}$$

◦ Find the determinant; this is also called the characteristic polynomial of M.

$$-t^3 + 5t^2 + t - 5$$

◦ Solve; the eigenvalues are the roots of the characteristic polynomial.

$$\begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

• Select an Eigenvalue

$$1$$

• Subtract the eigenvalue times the identity matrix from M

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

• Calculate $A=M-t \cdot \text{Id}$

$$\begin{bmatrix} 0 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

• Solve the system of equations $AX=0$

$$\begin{bmatrix} 0 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 0 \end{bmatrix} \cdot X = 0$$

▫ Apply Gaussian Elimination

◦ Add -2 times row 1 to row 2

$$\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$$

Gauss Jordan Elimination

The [GaussJordanEliminationTutor](#) command now has an option to return detailed steps for finding Eigenvalues.

> *with(Student:-LinearAlgebra) :*

> $M := \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 3 & 2 & 5 \\ 0 & 2 & 1 & 5 \end{bmatrix} :$

> *GaussJordanEliminationTutor(M, output = steps)*

Gauss-Jordan Reduce

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 3 & 2 & 5 \\ 0 & 2 & 1 & 5 \end{bmatrix}$$

- Add -2 times row 1 to row 2

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 1 & 5 \end{bmatrix}$$

- Multiply row 2 by -1

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 1 & 5 \end{bmatrix}$$

- Add -2 times row 2 to row 1

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 1 & 5 \end{bmatrix}$$

- Add -2 times row 2 to row 3

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$

- Multiply row 3 by 1/5

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & \frac{3}{5} \end{bmatrix}$$

- Add -4 times row 3 to row 1

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{7}{5} \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & \frac{3}{5} \end{bmatrix}$$

- Add 2 times row 3 to row 2

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{7}{5} \\ 0 & 1 & 0 & \frac{11}{5} \\ 0 & 0 & 1 & \frac{3}{5} \end{bmatrix}$$

(9.1)

Share your solution

There's a connection to the new product Maple Learn as well. These commands can output a link to a Maple Learn document containing the solution steps. Maple Learn is a dynamic online environment for teaching and learning math, focused on high-school to second year university. For more about Maple Learn, visit <https://www.maplesoft.com/products/learn/>

> *with(Student-Calculus1) :*

> *cv := ShowSolution(Int(sin(x)², x), output = canvas) :*

> *DocumentTools:-Canvas:-ShareCanvas(cv)*

<https://learn.maplesoft.com/#/?d=COOROFHKHLBHPMHSIOFJCGHPPSARFJMSNOPFCRKKPSNOILITKLDMKNESLUCFNDRDKFT ELLNIKLMREJDFFJKJDGMSJUOMPKHKIRJR>

> *with(Student-LinearAlgebra) :*

> $M := \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 1 \end{bmatrix} :$

> *EigenvectorsTutor(M, output = link)*

<https://learn.maplesoft.com/#/?d=HTNRCGNJBSDQKOCNLFPPKFAGANHOHILKPSIOLTTPPGPHSBROQBGNPLFDJFFPJEMCMM GKLOHHJGODRNIKUESCIOMIJJTEFOHNUFFNG>

Where did I go wrong?

In Maple 2021, students can now solve an equation by entering the step-by-step solution to the problem themselves, and then asking Maple for feedback. The responsive feedback lets the student know whether or not the solution is correct, and if not, where they went wrong. The [SolvePractice](#) command generates an interactive application where a student can type in the steps to solve a given problem. Then, the student clicks the button, and the application analyzes their steps and provides feedback.

> *with(Grading) :*

> *SolvePractice(3·x³ + 20·x² = x·(-3·x² - 9·x - 13), x)*

Solve this equation

Check Page

$$3x^3 + 20x^2 = x(-3x^2 - 9x - 13)$$

$$3 \cdot x^3 + 20 \cdot x^2 = -3 \cdot x^3 - 9 \cdot x^2 - 13 \cdot x \quad \text{ok}$$

$$6 \cdot x^3 + 29 \cdot x^2 + 13 \cdot x = 0 \quad \text{ok}$$

$$x \cdot (6 \cdot x^2 + 29 \cdot x + 13) = 0 \quad \text{ok}$$

$$x = 0$$

Good job, this is a correct solution

$$6 \cdot x^2 + 29 \cdot x + 13 = 0 \quad \text{ok}$$

$$(3 \cdot x + 13) \cdot (2 \cdot x + 1) = 0 \quad \text{ok}$$

$$3 \cdot x + 13 = 0 \quad \text{ok}$$

$$x = -\frac{13}{3}$$

Good job, this is a correct solution

$$2 \cdot x + 1 = 0 \quad \text{ok}$$

$$x = -\frac{1}{2}$$

Good job, this is a correct solution

Great! You found all of the correct

> New Math Entry Box

These practice-with-feedback sheets can be deployed to the web via Maple Learn:

> $SolvePractice(3 \cdot x^3 + 20 \cdot x^2 = x \cdot (-3 \cdot x^2 - 9 \cdot x - 13), x, output = link)$

<https://learn.maplesoft.com/#/?d=BHPKHJMJLGDJJHFTPOKRMLDKAULUCTBPPRHFDIEIANLUCIBOGRATCOHNAOKGMGLULONLAPLKENFUBUFIBMGGHUIJDNJSGFDJBSFK>